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Gravitomagnetic Barnett effect

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Abstract Using the linearized theory of general relativity, the gravitomagnetic analogue of the Barnett effect is derived Further theoretical and experimental investigations are recommended, due to the macroscopic values of the equivalent gravitomagnetic field involved in this effect, and to the constraints which would appear on quantum theories of gravity, currently under development, in case of non-detection of the predicted phenomena

Keywords Barnett effect gravitomagnetism, magnetization

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1. Linearized theory of general relativity

By linearizing Einstein's general relativity equations, gravitational analogues to the electric and magnetic fields are derived, named the *gravitoelectric* $\boldsymbol{E}_g$, and the *gravitomagnetic* $\boldsymbol{B}_g$ fields respectively [1]. Due to this similarity between gravitation and electromagnetism, it is possible to convert one field into the other by applying certain conversion constants [2]. Now, if we have a certain mass distribution and flow, all that is necessary is to find a similar charge and current distribution in electromagnetic texts. We then use the formulas derived for the electric and magnetic fields and make the substitutions in the electromagnetic formulas to obtain the respective gravitational analogue.

Once we have calculated the fields generated by the mass density and currents, we can calculate the forces on a particle of mass m by a force equation that is analogous to the Lorentz force equation,

$$\boldsymbol{F} = -m\nabla\phi - m\frac{\partial \boldsymbol{A}_g}{\partial t} + m\boldsymbol{v}\times\left(\nabla\times\boldsymbol{A}_g\right), \tag{1}$$

$$\boldsymbol{F} = m\boldsymbol{E}_g + m\boldsymbol{v}\times\boldsymbol{B}_g\,, \tag{2}$$

where ϕ is the gravitational scalar potential (m^2/s^2), $\boldsymbol{A}_g$ is the gravitomagnetic vector potential (m/s), m and v are respectively the mass and the speed of the test particle. Further, the gravitoelectric field $\boldsymbol{E}_g$ and the gravitomagnetic field $\boldsymbol{B}_g$ are given in the units m/s^2 and rad/s respectively.

If the test body is spinning and has an angular momentum $\boldsymbol{L}$, then the torque on it due to the gravitomagnetic field $\boldsymbol{B}_g$ will be by analogy (note that in the classical context, the Landé factor $g = 1$)

$$\boldsymbol{N} = \frac{1}{2}\boldsymbol{L}\times\boldsymbol{B}_g\,. \tag{3}$$

It should be emphasized that the previous discussion is approximate and is presented merely to provide a simple tool with which to make estimates and identify gravitational analogues of well known electromagnetic phenomena. By linearizing Einstein's equations, the following assumptions have been made :

(i) all motions are much slower than the speed of light to neglect special relativity,

(ii) the kinetic or potential energy of all bodies being considered is much smaller than their mass energy to neglect additional space curvature effects,

(iii) the distance between objects is not so large that we have to take retardation into account.

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We will show in the following, how this analogy can be used to calculate the gravitational analogue of the magnetic Barnett effect.

2. Gravitomagnetic Barnett Effect

In 1915, S J Barnett [3] observed that a body of any substance (initially unmagnetized) set into rotation, becomes the seat of a uniform intrinsic magnetic field parallel to the axis of rotation, and proportional to the angular velocity If the substance is magnetic, magnetization results, otherwise not This physical phenomenon is referred to as *magnetization by rotation* or as the *Barnett effect* The magnetization acquired by the material due to its rotation is the same as the one it would acquire if it would be submitted to an equivalent magnetic field [4,5] B_{equi} given by

$$B_{equi} = -\frac{1}{g}\frac{2m}{e}\Omega \tag{4}$$

where g is the Lande factor, and Ω is the angular velocity of the rotating material

Using the isomorphism between gravitation and electromagnetism derived above from general relativity, we are now able to compute the gravitational analogue of the magnetic Barnett effect Therefore in eq (4), we have to substitute B and e by B_g and m respectively and end with

$$B_{g\,equi} \qquad \Omega \tag{5}$$

Eq (5) tells us that any substance (initially ungravitomagnetized) set into rotation (see Figure 1), becomes the seat of a uniform intrinsic gravitomagnetic field parallel to the axis of rotation, and proportional to the angular velocity If the substance is gravitomagnetic, gravitomagnetization results, otherwise not This physical phenomenon shall be referred to as *gravitomagnetization by rotation* or as the *gravitomagnetic Barnett effect.*

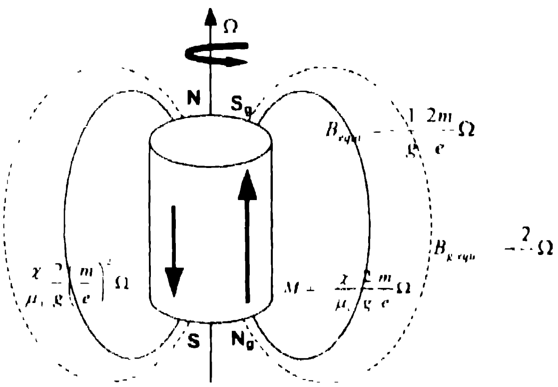

Figure 1. A rotating non-ferromagnetic cylinder with its associated magnetic and gravitomagnetic equivalente fields and their respective magnetization and gravitomagnetization vectors

The gravitomagnetic moment at the quantum level is defined as $m_g = \frac{g}{2}I$. Due to its definition, the Landé factor g is a pure number (no dimension) and shall have the same value for gravitomagnetic and magnetic phenomena.

$$g = g_{GM} \tag{6}$$

Nevertheless, further research is needed to firmly establish this result. The only experiments using spin alignment to predict gravitational interactions are described in patents by H W Wallace [6,7] His macroscopic observations, supporting our simple derivation, however have never been reproduced or seriously investigated in the literature up to now

3. Discussion

In nature, every electromagnetic field is associated with a gravitic-gravitomagnetic field depending on the charge-to mass ratio from the source particle [2]

$$B_g = -\frac{\mu_{0g}}{\mu_0}\frac{m}{e}B, \tag{7}$$

$$\gamma = \frac{\mu_{0g}}{\mu_0}\frac{m}{e}E \tag{8}$$

However, due to the possibility of having neutral matter only the gravitomagnetic field can exist without an associated electromagnetic field Magnetization and gravitomagnetization generally appear together [2]

$$M_g = \frac{m}{e}M. \tag{9}$$

If magnetization and gravitomagnetization results from the application of a magnetic field to the body, the relation between the magnetic and gravitomagnetic susceptibility is

$$\chi = \chi_g. \tag{10}$$

If magnetization and gravitomagnetization results from the application of a gravitomagnetic field to the body, the relation between the magnetic and gravitomagnetic susceptibility is :

$$\frac{\chi_g}{\chi} = \frac{\mu_{0g}}{\mu_0}\left(\frac{m}{e}\right) \tag{11}$$

(i) The magnetization and gravitomagnetization one obtains through the magnetic and gravitomagnetic Barnett effect (for non-ferromagnetic materials) are respectively :

$$M = \frac{\chi}{\mu_0}B_{equi} = -\frac{\chi}{\mu_0}\frac{2}{g}\frac{m}{e}\Omega \tag{12}$$

$$M_g = \frac{\chi_g}{\mu_{0g}}B_{g\,equi} = \frac{\chi_g}{\mu_{0g}}\frac{2}{g}\Omega \tag{13}$$

The magnetization and gravitomagnetization one obtains through the rotation of a (non-ferromagnetic) body by aligning mechanically the angular momentum of the gyrostats

$(\varepsilon = I\,\Omega)$ is similar to the magnetization and the gravitomagnetization one obtains by applying an external magnetic field $\boldsymbol{B}_{equi}$ (which will align the magnetic moments of the gyrostats, $\varepsilon = \mu\, \boldsymbol{B}_{equi}$) or by applying an external gravitomagnetic field $\boldsymbol{B}_{g\,equi}$ (which will align the gravitomagnetic moments of the gyrostats, $\varepsilon = \mu_{gm}\, \boldsymbol{B}_{g\,equi}$) on the same body at rest (with no rotation)

(ii) Let us suppose the body is at rest and non-rotating. We apply a magnetic field $B_{equi} = \frac{1}{g}\frac{2m}{e}\Omega$ on it. What will be the magnetization and gravitomagnetization resulting from this process?

$$\boldsymbol{M} = \frac{\chi}{\mu_0}\frac{2}{g}\frac{m}{e}\Omega . \tag{14}$$

Using eqs (9) and (10), we can compute the associated gravitomagnetic vector

$$\boldsymbol{M}_g = \frac{\chi}{\mu_0}\frac{2}{g}\left(\frac{m}{e}\right)\Omega \tag{15}$$

Using eq. (7), we see that this gravitomagnetization vector will be associated with a gravitomagnetic field

$$\boldsymbol{B}_g = \frac{\mu_{0g}}{\mu_0}\frac{2}{g}\left(\frac{m}{e}\right)\Omega \tag{16}$$

which is much weaker than $\boldsymbol{B}_{g\,equi}$.

(iii) Let us now consider the same body at rest and non-rotating. We apply an external gravitomagnetic field of intensity $\boldsymbol{B}_{g\,equi}$. Let us compute the resulting magnetization and gravitomagnetization

$$\boldsymbol{M}_g = \frac{\chi_g}{\mu_{0g}}\frac{2}{g}\Omega . \tag{17}$$

Using eq (11), we can write eq (17) in the following form

$$\boldsymbol{M}_g = \frac{\chi}{\mu_0}\frac{2}{g}\left(\frac{m}{e}\right)\Omega \tag{18}$$

Now applying the inverse of eqs (9) to (18) we get

$$\boldsymbol{M} = -\frac{\chi}{\mu_0}\frac{2}{g}\frac{m}{e}\Omega \tag{19}$$

Using the inverse of eq. (7), we can see that this magnetization vector will be associated with a magnetic field

$$\boldsymbol{B} = \frac{\mu_0}{\mu_{0g}}\frac{2}{g}\frac{e}{m}\Omega \tag{20}$$

which is much higher than $\boldsymbol{B}_{equi}$.

Comparing the magnetization and gravitomagnetization processes in (i–iii), we see that we always end with the same magnetization and gravitomagnetization vectors despite the fact that the associated couple of magnetic and gravitomagnetic fields have not at all the same value. Therefore, the three different processes are equivalent from the point of view of magnetization and gravitomagnetization but are not at all equivalent from the point of view of the intensity of the magnetic and gravitomagnetic fields involved in the three different processes.

Therefore, the gravitomagnetic Barnett effect would be a physical effect which could involve large equivalent gravitomagnetic fields despite the fact of being associated with bodies having an extremely small gravitomagnetization

4. Conclusion

A gyrogravitomagnetic experiment gives no information at all concerning the process of gravitomagnetization of the rotating substance. However, the presence of the Landé factor g in eq (5) indicates that the use of an angular velocity to generate an equivalent gravitomagnetic field could involve a quantum process.

The value of the Landé factor in the context of gravitomagnetism, the value of the gravitomagnetic susceptibility of different materials, and the possible processes of gravitomagnetization [2,8] of different substances shall be evaluated. In the case of non-detection of the predicted effect, we might conclude that the concept of quantum gravitomagnetic moment does not make sense. This would impose important constraints on the theories of quantized gravity being currently elaborated

Therefore, further theoretical and experimental investigations are required to confirm or not the predicted gravitomagnetic Barnett effect. This investigation is justified amongst other reasons, by the macroscopic value of the equivalent gravitomagnetic field involved in the effect.

References

[1] R L Forward *Proc of the IRE, May* p892 (1961)

[2] M Tajmar and C J de Matos *Journal of Theoretics* **3** (Los Alamos Archive gr-qc/0003011) February (2001)

[3] S J Barnett *Phys Rev* 6 239 (1915)

[4] L F Bates *Modern Magnetism* (Cambridge Cambridge University Press) **Vol 2** p243 (1963)

[5] S Chikazumi *Physics of Magnetism* (New York John Wiley and Sons) p39 (1964)

[6] H W Wallace *Method and Apparatus for Generating a Secondary Gravitational Force Field* (United States Patent, US 3 626 605) December 14 (1971)

[7] H W Wallace *Method and Apparatus for Generating a Dynamic Force Field* (United States Patent, US 3 626 606, December 14) (1971)

[8] R L Forward *Am J Phys March* p166 (1963)